Generalized Unidirectional Pulse Propagation Equations: Treatment of Waveguides

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Abstract

The generalized Unidirectional Pulse Propagation Equation (gUPPE) [1] can handle simulation regimes with the following attributes:
1) Structures with strong refractive index contrasts.
2) Directional long-distance wave propagation.
3) Strong waveform reshaping (time and space).
4) Extreme spectral dynamics; resulting spectra often encompass more than an octave in frequency.

A capillary waveguide is studied with gUPPE and compared to the typical method, which expands the electric field into approximate leaky modes.

Remaining demonstrations simplify the gUPPE in order to improve computational efficiency specifically for waveguides. The central approximation relies on generating a boundary condition at the core-cladding interface, allowing the cladding itself to be entirely neglected.

gUPPE: Derivation Overview

Assumptions:

- No free charges or current
- Short time scale: ionized particles remain close and local charge = 0
- Material interfaces parallel to propagation direction

Transverse part of wave equation split into **LINEAR**

$$\hat{L}E_\perp = \frac{\omega^2}{c^2}\epsilon(r_\perp,\omega)E_\perp + \nabla_\perp^2 E_\perp + \nabla \frac{1}{\epsilon} E_\perp \cdot \nabla_\perp \epsilon$$

And **NONLINEAR** terms

$$\hat{N}(E) = \frac{\omega^2}{\epsilon_0 c^2}P(E) + \nabla \frac{1}{\epsilon_0 \epsilon}\nabla \cdot P(E)$$

Forward & Backward Propagation Equations

EXACT solution, given the above assumptions

$$\partial_z E_\perp^F = +i\sqrt{\hat{L}}E_\perp^F + \frac{i}{2\sqrt{\hat{L}}}\hat{N}_\perp[E^F + E^B]$$

$$\partial_z E_\perp^B = -i\sqrt{\hat{L}}E_\perp^B - \frac{i}{2\sqrt{\hat{L}}}\hat{N}_\perp[E^F + E^B]$$

Employ amplitudes A that evolve only if nonlinearity is present

$$E_\perp^F = e^{+i\sqrt{\hat{L}}z}A_\perp^F \; ; \; E_\perp^B = e^{-i\sqrt{\hat{L}}z}A_\perp^B$$

Generalized UPPE

Unidirectional Approximation

$$\hat{N}_\perp\left[e^{+i\sqrt{\hat{L}}z}A^F + e^{-i\sqrt{\hat{L}}z}A^B\right] \approx \hat{N}_\perp\left[e^{+i\sqrt{\hat{L}}z}A^F\right]$$

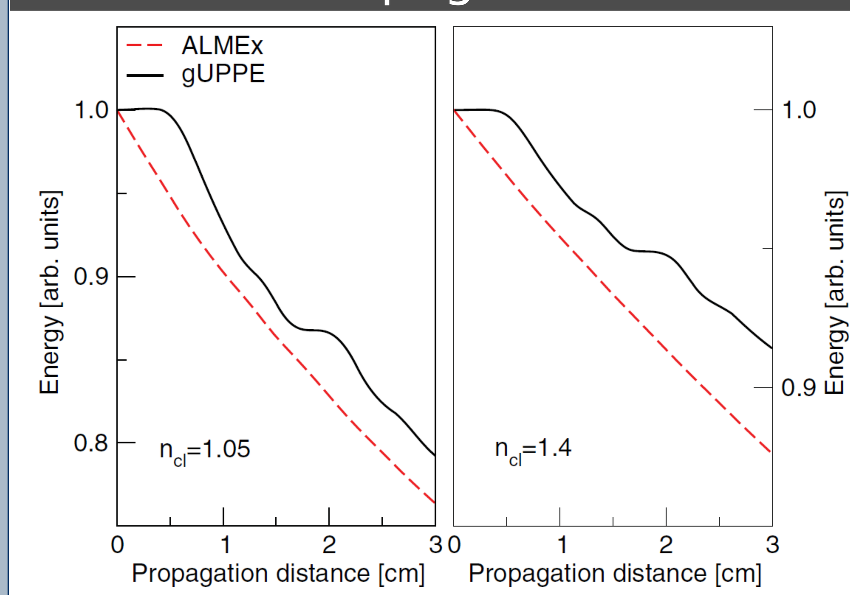
$$\partial_z A_\perp^F(r_\perp,\omega,z) = \frac{+i}{2\sqrt{\hat{L}}}e^{-i\sqrt{\hat{L}}z}\hat{N}_\perp\left[e^{+i\sqrt{\hat{L}}z}A^F\right]$$

gUPPE Demonstration

Pressurized capillary [2] waveguide: mid-infrared pulse.

- Inner radius r = 200 μm
- Ar pressurized to 20 atm
- Core-cladding index difference n_{cl}

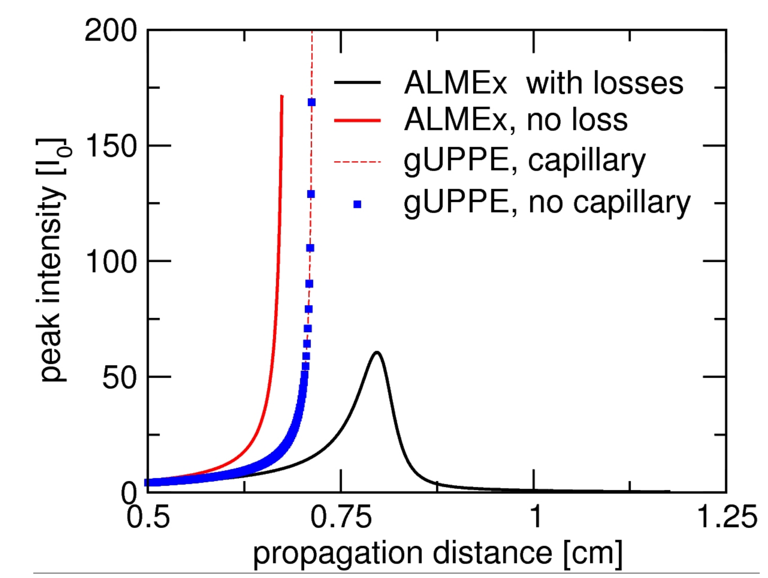
Linear Propagation



gUPPE compared to usual method of Approximate Leaky Mode Expansion (**ALMEx**).

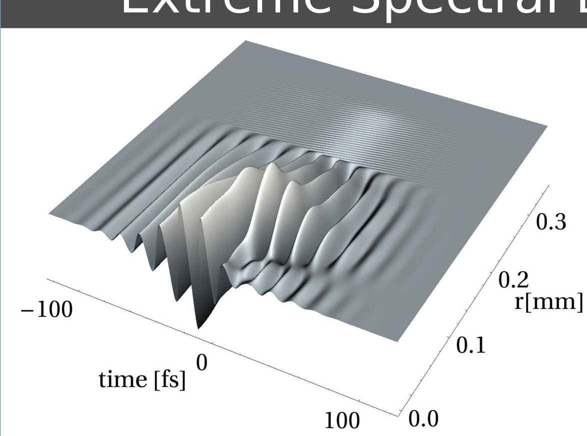
Energy loss incorrectly predicted by **ALMEx** due to spatially uniform loss based on propagation constants ascribed to a set of orthonormal modes.

Self Focusing



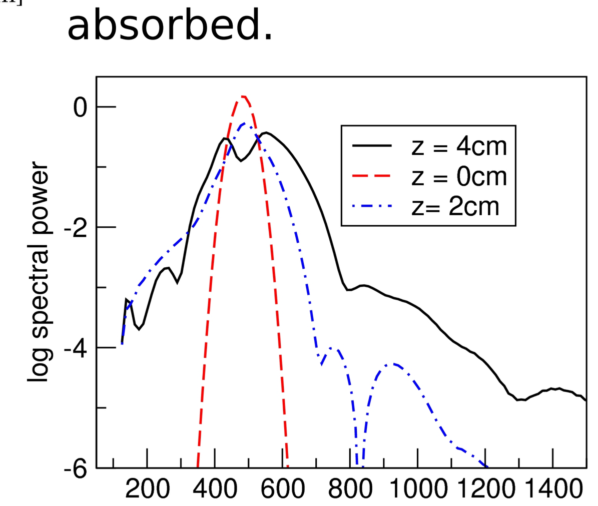
Peak intensity explodes under self focusing collapse. Initial beam waist smaller than capillary core diameter — no significant interaction with cladding. Spatially uniform losses applied blindly with **ALMEx** increase the threshold for self-focusing collapse.

Spatiotemporal Reshaping & Extreme Spectral Broadening



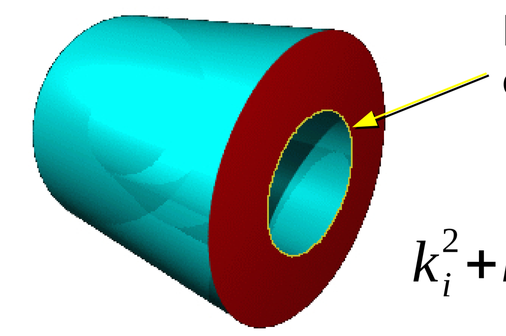
Excitation of **high-order** modes as above. Trailing edge of pulse in on-axis region depleted by plasma. Outgoing waves absorbed.

Propagation regime at high pressure akin to **fs** filamentation with **supercontinuum generation** reflecting extreme nonlinear evolution.



gUPPE-b: Leaky Waveguides

Goal: propagation confined to core; neglect cladding (allows efficient computation at small kR values)



Boundary condition derived at core-cladding interface [3]

Dispersion relations

$$k_i^2 + k_{||}^2 = \frac{\omega^2 n^2}{c^2} \; ; \; k_o^2 + k_{||}^2 = \frac{\omega^2 n_{cl}^2}{c^2}$$

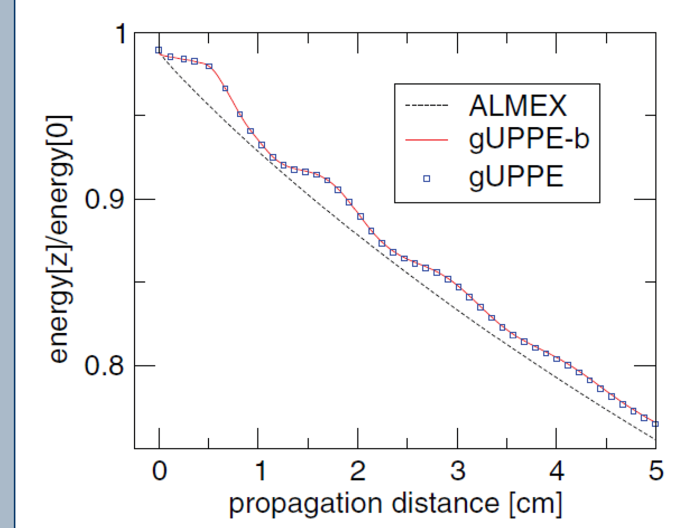
Paraxial approximation: $k_i \ll k_o \longrightarrow k_o = \frac{\omega}{c}\sqrt{n_{cl}^2 - n^2}$

New boundary condition

The field at the grid point just inside the core E_i serves as a semi-transparent boundary.

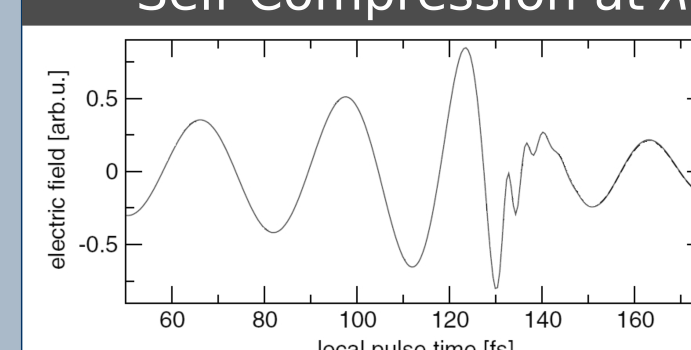
$$E_i^{TM} = \frac{4E_{i-1} - E_{i-2}}{3 - 2ik_o\Delta x\, n^2/n_{cl}^2}$$

$$E_i^{TE} = \frac{4E_{i-1} - E_{i-2}}{3 - 2ik_o\Delta x}$$



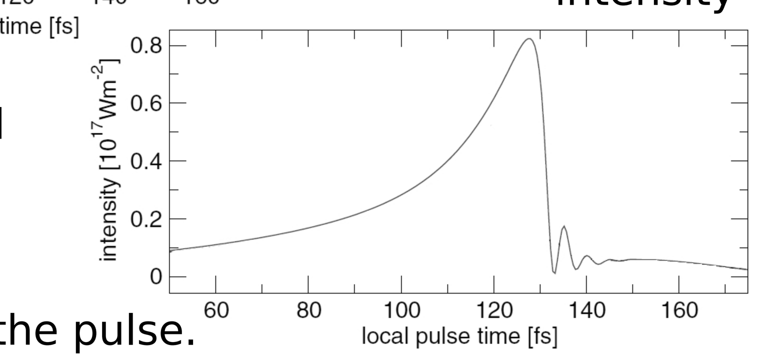
Method validation
Linear regime: Pulse focused at the entrance of the capillary waveguide.

Self Compression at λ = 8 μm



Electric field waveform

Cycle-averaged intensity

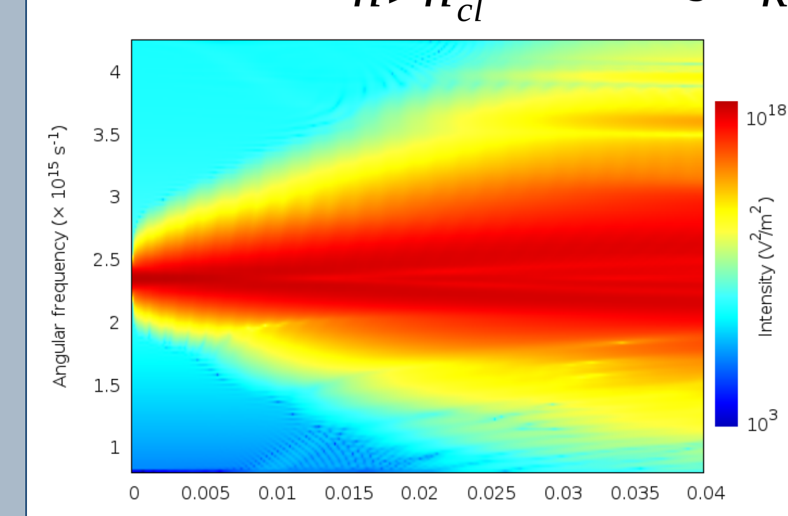


Stronger chirp and high-frequency components are "localized" in the trailing portion of the pulse.

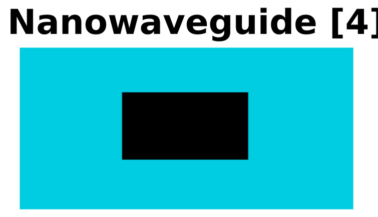
gUPPE-b: High-contrast Waveguides

Same boundary condition as above, except

$n > n_{cl}$ meaning $k_o \rightarrow ik_o$



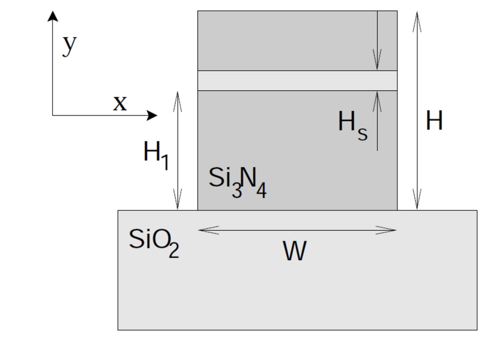
Nanowaveguide [4]



Extreme spectral **broadening** just below critical power for self focusing.

Core-confined BPM: Slot Waveguide

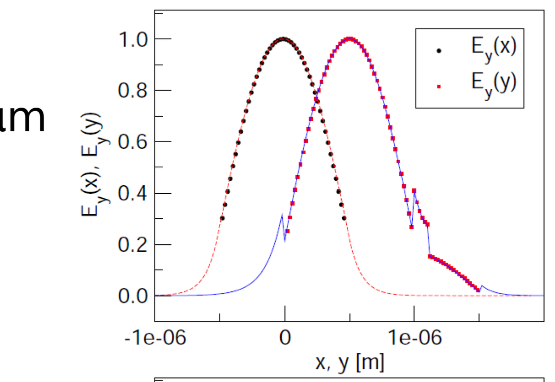
Continuous-wave regime



Silicon nitride slot waveguide for supercontinuum generation on chip.

Fundamental TM mode field calculation

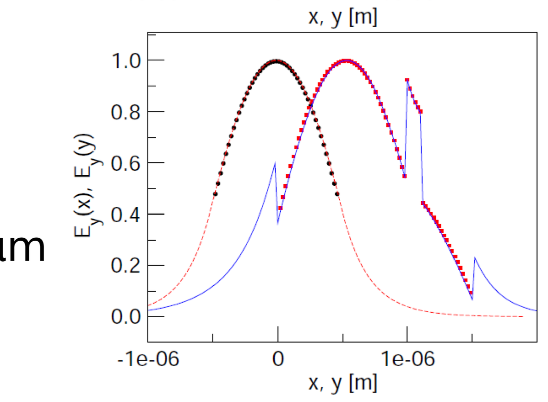
λ = 1μm



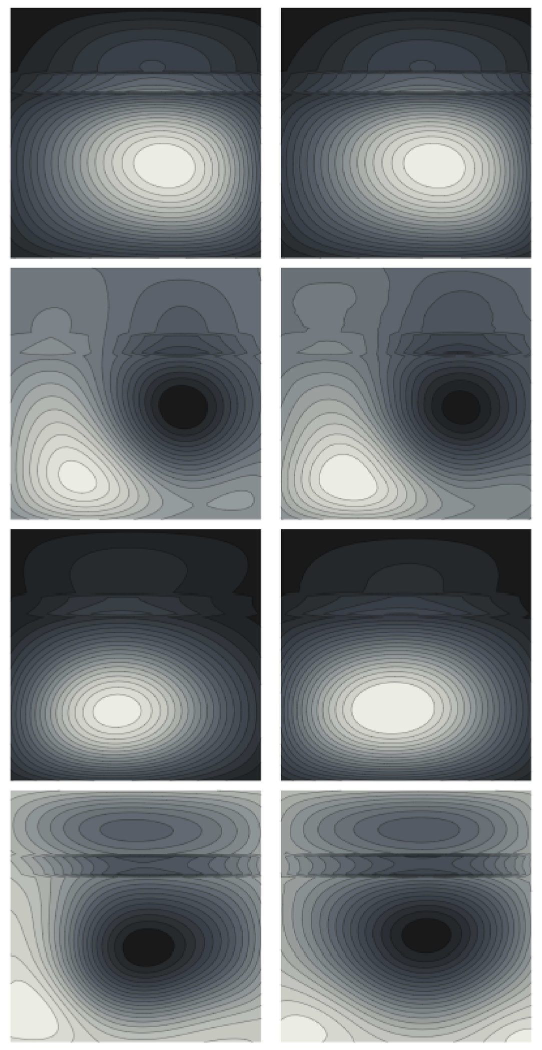
Lines: iterated Crank Nicolson method (iCN)

Symbols: ccBPM

λ = 2μm



iCN ccBPM



Z = 0: Initial condition consists of arbitrary modal superposition at λ = 1μm

Propagation distance Z = 2.5μm

Z = 5.0 μm

Z = 7.5 μm

Propagation distance over the "period" of mode beating

Excellent agreement even when only simulating the waveguide core

References

[1] J. Andreasen and M. Kolesik, "Nonlinear propagation of light in structured media: Generalized unidirectional pulse propagation equations," Phys. Rev. E **86**, 036706 (2012).

[2] T. Popmintchev, et al., "Bright Coherent Ultrahigh Harmonics in the keV X-ray Regime fromMid-Infrared Femtosecond Lasers," Science **336**, 1287 (2012).

[3] J. Andreasen and M. Kolesik, "Midinfrared femtosecond laser pulse filamentation in hollow waveguides: A comparison of simulation methods," Phys. Rev. E **87**, 053303 (2013).

[4] M. A. Foster et al., "Broad-band optical parametric gain on a silicon photonic chip," Nature **441**, 960 (2006).

Acknowledgments

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