Nonlinear propagation of light in structured media: generalized unidirectional pulse propagation equations

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Abstract

The unidirectional pulse propagation equations (UPPE) [1] are generalized to structures characterized by strong refractive index differences and material interfaces.

Introduction

UPPE has provided a theoretical basis for computer-aided investigations into dynamics of **high-power ultrashort laser pulses** and have been successfully utilized for almost a decade. However, they are restricted to applications in bulk media or to simple waveguide geometries in which only a few guided modes can approximate the propagating waveform.

Here, we describe a generalization of the UPPE, which can be applied to nonlinear structured media with strong differences between refractive indices of the constituent materials.

Simulation Regimes

1) Structures with strong refractive index contrasts.

2) Directional long-distance wave propagation.

3) Strong waveform reshaping, both in time and space.

4) Extreme spectral dynamics, with resulting spectra often encompassing more than an octave in frequency.

Approach

- Propagation separated into forward and backward parts. Full transverse structure included implicitly.
- Unidirectional propagation approximation applied
- Resulting equations transformed into a system analogous to bulk-medium UPPE.
- Linear propagator treated by techniques developed for wide-angle BPM (WA-BPM) [2-4].
- Nonlinear interactions treated by ordinary differential equation (ODE) libraries, the same way it has been done with UPPE-based simulators [5].

Starting Point

Wave Equation

$$\nabla\nabla\cdot\vec{E}-\nabla^2\vec{E}=\frac{\omega^2}{c^2}\left(\epsilon\vec{E}+\frac{1}{\epsilon_0}\vec{P}\right)$$

Divergence Equation

$$\nabla\cdot\vec{D}=0$$

Forward and Backward Propagation Equations

$$\partial_z E_\perp^F=+i\sqrt{\hat{L}}E_\perp^F+\frac{i}{2\sqrt{\hat{L}}}\hat{N}_\perp[E^F+E^B]$$

$$\partial_z E_\perp^B=-i\sqrt{\hat{L}}E_\perp^B-\frac{i}{2\sqrt{\hat{L}}}\hat{N}_\perp[E^F+E^B]$$

Employ amplitudes, which only evolve if nonlinearity is present

$$E_\perp^F=e^{+i\sqrt{\hat{L}}z}A_\perp^F\;;\;E_\perp^B=e^{-i\sqrt{\hat{L}}z}A_\perp^B$$

Linear and Nonlinear Operators

Assume:

- No free charges or current
- Short time scale (femtosecond pulses) so that ionized particles remain close and local charge = 0
- Material interfaces parallel to direction of propagation

$$\nabla\cdot\vec{E}=\frac{1}{\epsilon}\vec{E}_\perp\cdot\nabla_\perp\epsilon+\frac{1}{\epsilon_0\epsilon}\nabla\cdot\vec{P}$$

Transverse part of wave equation above split into **LINEAR**

$$\hat{L}E_\perp=\frac{\omega^2}{c^2}\epsilon(r_\perp,\omega)E_\perp+\nabla_\perp^2E_\perp+\nabla\frac{1}{\epsilon}E_\perp\cdot\nabla_\perp\epsilon$$

And **NONLINEAR** terms

$$\hat{N}(E)=\frac{\omega^2}{\epsilon_0c^2}P(E)+\nabla\frac{1}{\epsilon_0\epsilon}\nabla\cdot P(E)$$

Generalized UPPE

Unidirectional Approximation

$$\hat{N}_\perp\left[e^{+i\sqrt{\hat{L}}z}A^F+e^{-i\sqrt{\hat{L}}z}A^B\right]\approx\hat{N}_\perp\left[e^{+i\sqrt{\hat{L}}z}A^F\right]$$

$$\partial_z A_\perp^F(r_\perp,\omega,z)=\frac{+i}{2\sqrt{\hat{L}}}e^{-i\sqrt{\hat{L}}z}\hat{N}_\perp\left[e^{+i\sqrt{\hat{L}}z}A^F\right]$$

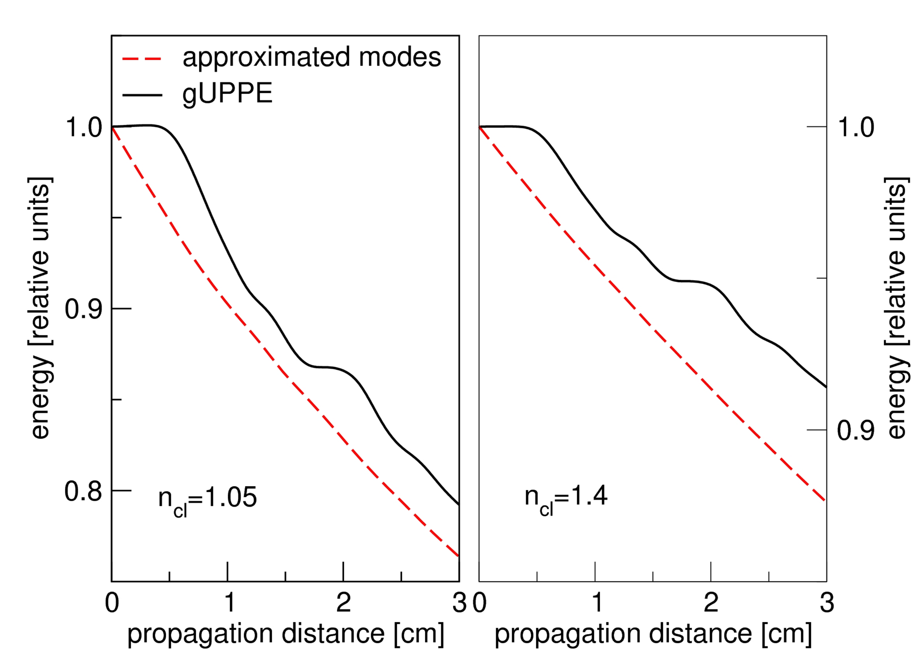
Geometry for Demonstration [6]

Pressurized Capillary waveguide excited by a mid-infrared pulse.

- Inner radius r = 200 μm
- Pressurized to 20 atm with Ar
- Index difference between core and cladding: n_{cl}

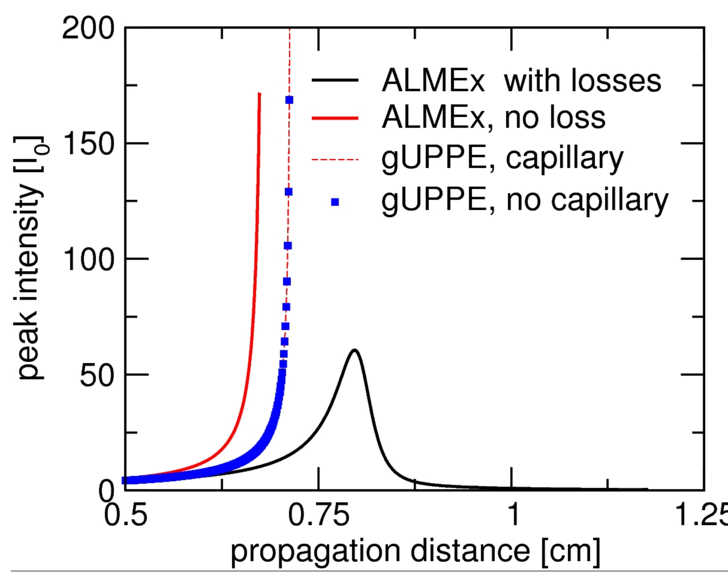
Linear Propagation

Generalized UPPE (**gUPPE**) compared to usual method of Approximate Leaky Mode Expansion (**ALMEx**).



Energy loss experienced by initially collimated, pulsed Gaussian-beam with the beam waist smaller than the capillary inner diameter. **gUPPE** includes proper modelling of loss due to light leaking into the cladding. **ALMEx** incorrectly predicts loss since it is based on complex-valued propagation constants ascribed to orthonormal set of modes (spatially uniform loss).

Self Focusing



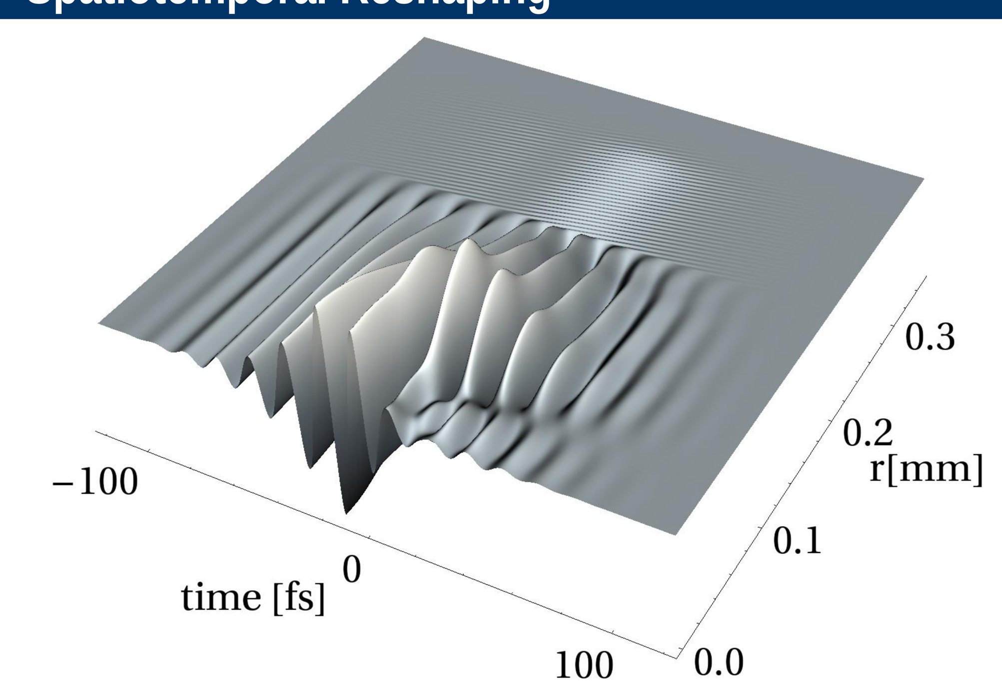
Self-focusing in the nonlinear regime in the waveguide causes an **explosion** of localized intensity above a critical power and at a critical propagation distance.

The initial beam waist is smaller than the capillary inner diameter so it does not significantly interact with the cladding.

If losses due to high-transverse-wavenumber modes are applied blindly with **ALMEx**, collapse becomes impossible.

gUPPE correctly accounts for the spatial profile of the beam.

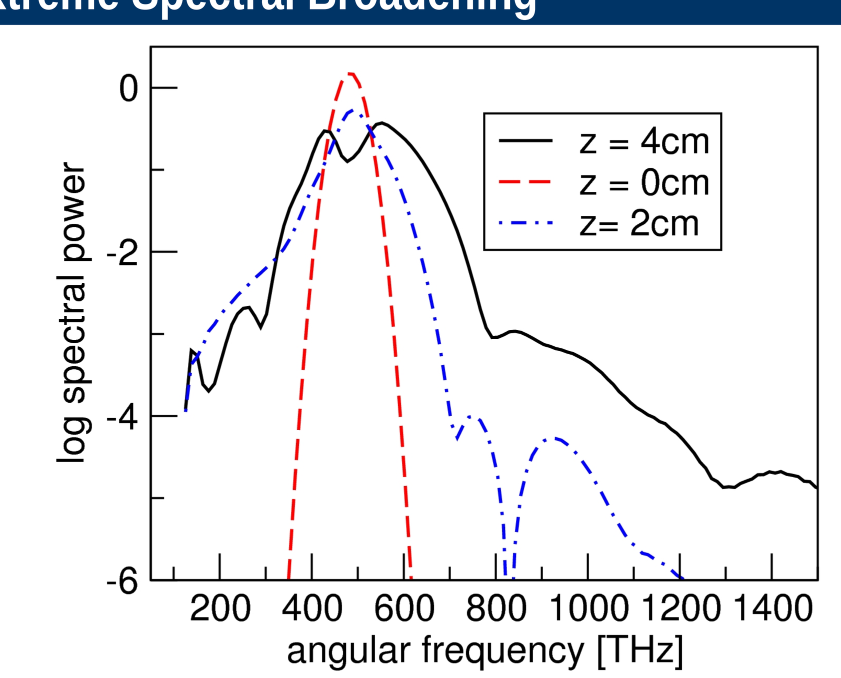
Spatiotemporal Reshaping



Significant excitation of high-order modes. Such modes have higher loss, but a superposition of them confines the total waveform to the interior of the waveguide. Actual loss is minimal. gUPPE correctly simulates this, but ALMEx cannot.

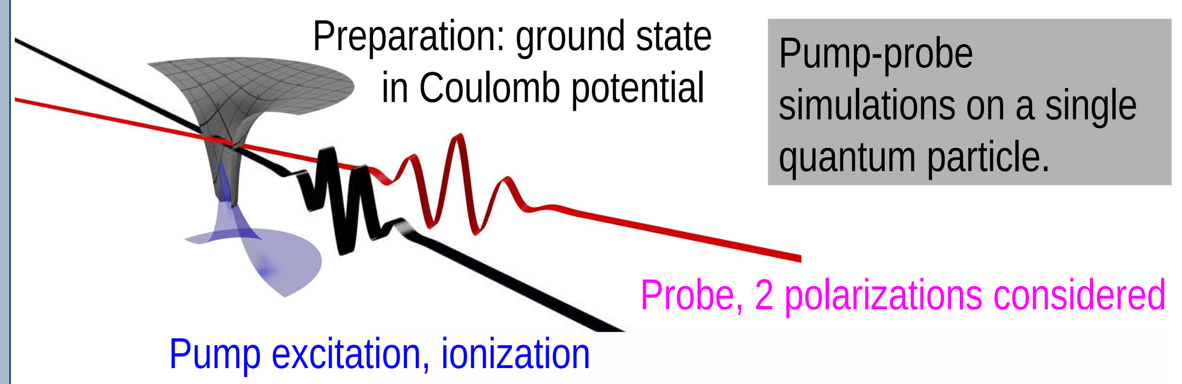
Trailing edge of pulse in on-axis region depleted by generated plasma. Outgoing waves at r > 0.2mm absorbed by PML.

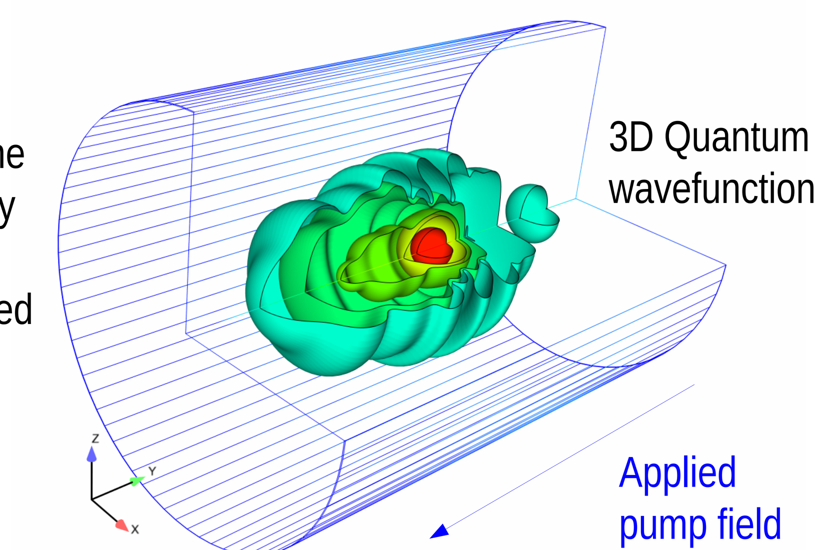
Extreme Spectral Broadening



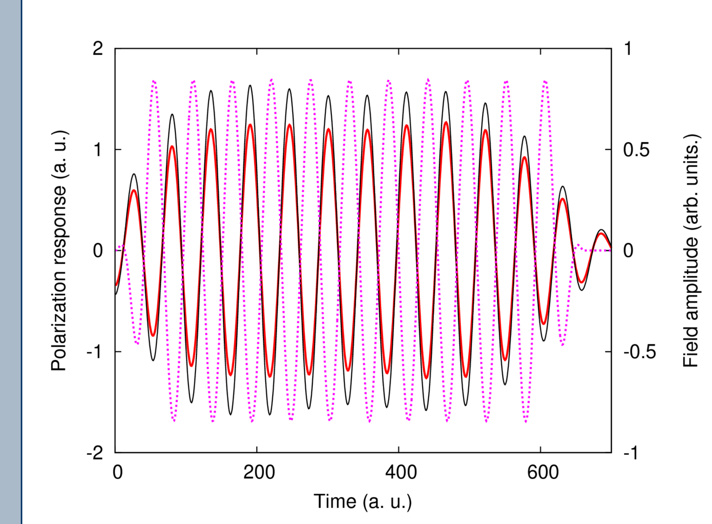
Propagation regime at high pressure is akin to femtosecond filamentation with the concomittant **supercontinuum** generation reflecting extreme nonlinear evolution.

Simulation Framework Applied to Quantum Problem





A pump of varying strengths excites the particle and partially ionizes it. Freed electrons are created and probed by an additional field. By using probe field polarizations **parallel & perpendicular** to the pump field, we investigate the freed electron response in different directions—testing its isotropy.

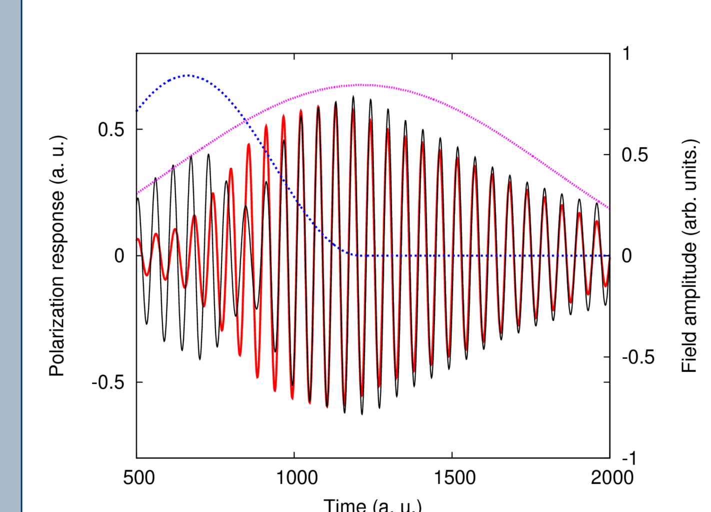


δ(t)-function Pump field

Freed electrons exhibit a characteristically out-of-phase response to probe (note **anisotropy**).

Response to Probe ||

Response to Probe ⊥



Realistic Pump-Probe

Overlapping pump-probe

Pump ~ 15 fs pulse

~ 10^{14} W/cm^2

Freed electrons are probed once the pump decays. Again, note **anisotropy.**

References

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Acknowledgments

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